5B Quiz 5: 7.1, 7.2 and 7.4i

1) $\int \cos^2(3x)dx$ $u=3x$, $du=3dx$

$= \frac{1}{3}\int \cos^2 u\,du$

For this integral, trig identity

$\int \cos^2 u\,du = \int \frac{1+\cos 2u}{2}du$

$= \frac{1}{2}\int(1+\cos 2u)du$

$= \frac{1}{2}(u+\frac{1}{2}\sin u)+C$

$= \frac{1}{2}u+\frac{1}{4}\sin 2u+C$

or use formula

$\boxed{\int \cos^2 u\,du = \frac{1}{2}u+\frac{1}{4}\sin 2u+C \quad \int \sin^2 u\,du = \frac{1}{2}u-\frac{1}{4}\sin 2u+C}$

$= \frac{1}{3}(\frac{1}{2}u+\frac{1}{4}\sin 2u)+C$

$= \frac{1}{3}(\frac{1}{2}\cdot 3x+\frac{1}{4}\sin(2\cdot 3x))+C$

$= \boxed{\frac{1}{2}x+\frac{1}{12}\sin 6x+C}$

2) $\int_0^{\pi/3} \sin(x)\ln(\cos x)dx$ $u=\cos x$, $du=-\sin x\,dx$

or IBP

$u=\ln(\cos x)$ $dv=\sin x\,dx$

$du=\frac{-\sin t}{\cos t}dx$ $v=-\cos x$

$\vdots$

$= -\int_1^{1/2}\ln u\,du$

$= \int_{1/2}^{1}\ln u\,du$

IBP

$u=\ln u$ $dv=du$

$du=\frac{1}{u}du$ $v=u$

$\int \ln u\,du = u\ln u-\int du$

$= u\ln u-u+C$

$\boxed{\int \ln u = u\ln u-u+C}$

$= u\ln u-u\Big]_{1/2}^{1}$

$= -1-(\frac{1}{2}\ln\frac{1}{2}-\frac{1}{2})$

$= \boxed{-\frac{1}{2}-\frac{1}{2}\ln\frac{1}{2}}$

3) $\int \tan^{-1}(3\theta)d\theta$ IBP: $u=\tan^{-1}(3\theta)$ $dv=d\theta$

$du=\frac{3}{1+9\theta^2}d\theta$ $v=\theta$

$= \theta\tan^{-1}(3\theta)-\int\frac{3\theta}{1+9\theta^2}d\theta$

$u=1+9\theta^2$

$du=18\theta\,d\theta$

$\frac{1}{18}\int\frac{3}{u}du$

$\frac{1}{6}\ln|u|$

$\frac{1}{6}\ln(1+9\theta^2)$

$= \boxed{\theta\tan^{-1}3\theta-\frac{1}{6}\ln(1+9\theta^2)+C}$

4) $\int \sin(\ln x)\,dx$

u-sub: $u = \ln x$ $\quad x = e^u$

$du = \frac{1}{x}dx$

$x\,du = dx$

$= \int x \sin u\,du = \int e^u \sin u\,du$. Now I.B.P. (see 7.1 example #4)

$= \frac{1}{2}e^u(\sin u - \cos u) + C$

$= \frac{1}{2}e^{\ln x}(\sin(\ln x) - \cos(\ln x)) + C$

$= \boxed{\frac{1}{2}x\,(\sin(\ln x) - \cos(\ln x)) + C}$

5). Review 7.4i (Partial Fraction Deomposition) as needed. Then find the partial fraction decomposition of :

$$\frac{x^2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$$

so $x^2 = A(x^2+1) + (Bx+C)(x-1)$

Equate coefficients

$$\left.\begin{array}{l} x^2 = Ax^2 \qquad\quad + A \\ \quad\;\; Bx^2 - Bx \\ \qquad\qquad + Cx \;\; - C \end{array}\right\} \Rightarrow \begin{cases} A+B=1 \\ -B+C=0 \\ A-C=0 \end{cases} \text{ solve.}$$

OR

shortcut

Let $x=1$ $\quad x^2 = A(x^2+1) + (Bx+C)(x-1)$

$1 = 2A \Rightarrow A = 1/2$

So $A = \frac{1}{2}$ $\quad B = \frac{1}{2}$ $\quad C = 1/2$

$$\boxed{\frac{1}{2(x-1)} + \frac{1}{2}\frac{(x+1)}{x^2+1}}$$